Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet answers for 2021-10-11

If you would like clarification on any problems, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to conceptual questions

Question 1. On each square, f attains its maximum value at the bottom-right corner. So the sample points to take are $(1,0),(2,0),(1,1),(2,1),(1,2),(2,2)$.

Question 2.

(a) This is impossible to integrate with respect to x, so we have to use $\mathrm{d}y\,\mathrm{d}x$.
(b) Either method is fine, though $\mathrm{d}y\,\mathrm{d}x$ would let you do it in a single integral.
(c) Either method is fine.
(d) It is practically impossible to solve $x = y - y^3$ for y in terms of x, so the only choice is to use the integration order $\mathrm{d}x\,\mathrm{d}y$.

Question 3. This integral is maximized if we take the region R to be exactly the region on which the integrand is nonnegative (or positive; it doesn't matter):

$$3 - x^2 + 2x - 4y^2 \geq 0$$

After completing the square, one can rewrite this as

$$(x-1)^2 + 4y^2 \leq 4$$

so we see it's a filled-in ellipse.

Question 4.

(a) Applying the hinted change of variables gives

$$\int_{-3}^{3} \arctan(x^3)\,\mathrm{d}x = \int_{3}^{-3} -\arctan(u^3)(-\,\mathrm{d}u) = -\int_{-3}^{3} \arctan(u^3)\,\mathrm{d}u$$

so this means the original integral must be equal to 0. Geometrically: $\arctan(x^3)$ is an odd function, so by integrating from -3 to 3, the area beneath the x-axis exactly cancels the area above it by symmetry.

(b) Switching the order of integration gives

$$\int_{-1}^{3} \int_{-\frac{1}{2}\sqrt{3-x^2+2x}}^{\frac{1}{2}\sqrt{3-x^2+2x}} e^{x^2+y^2} \sin y\,\mathrm{d}y\,\mathrm{d}x = \int_{-1}^{3} 0\,\mathrm{d}x = 0$$

where the inner integral evaluates to zero by the same logic as in part (a).

Answers to computations

Problem 1. The function to maximize/minimize is just $f(x,y,z) = z$. So our system of equations is

$$\begin{aligned} 0 &= 4\lambda + 2x\mu \\ 0 &= -3\lambda + 2y\mu \\ 1 &= 8\lambda - 2z\mu \\ 4x - 3y + 8z &= 5 \\ z^2 &= x^2 + y^2. \end{aligned}$$

The first two equations give $(6x + 8y)\mu = 0$, so either $\mu = 0$ or $3x + 4y = 0$. The former results in no solutions, as either of the first two equations implies $\lambda = 0$, but then the third equation is not satisfied. Hence we are left with the second case, and the system of equations

$$\begin{aligned} 3x + 4y &= 0 \\ 4x - 3y + 8z &= 5 \\ z^2 &= x^2 + y^2. \end{aligned}$$

After some algebra, we find the solutions $(x, y, z) = (-4/3, 1, 5/3)$ and $(4/13, -3/13, 5/13)$. The former is the highest point and the latter is the lowest point, because $5/3 > 5/13$.